

# Vacuum Instability in Anomaly Mediation Models with Massive Neutrinos

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## Abstract

We study the vacuum stability in the anomaly mediated supersymmetry (SUSY) breaking models with massive neutrinos. It is shown that, because of the seesaw-induced mass terms for neutrinos, the true vacuum has a large negative cosmological constant provided that the vacuum where we now live has an (almost) vanishing cosmological constant. Although the quantum transition into the true vacuum from our false vacuum is highly suppressed, the thermal transition at high temperatures may not be neglected because of the thermal excitations. However, we find that the thermal transition is, in fact, negligibly small and hence the anomaly mediation models are cosmologically safe. Thus, we conclude that the reheating temperature  $T_R$  could be very high (*e.g.*  $T_R \gg 10^{10} \text{ GeV}$ ) in the anomaly mediation models even with the seesaw-induced mass terms for neutrinos.

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## I. INTRODUCTION

Recent Superkamiokande experiments on atmospheric neutrinos [1] have presented very convincing evidence for the oscillation of  $\nu_\mu$  to  $\nu_\tau$  with a mass difference  $\delta m^2 \simeq 10^{-3} - 10^{-2} \text{ eV}^2$ . If neutrinos are indeed massive, the seesaw mechanism [2] is the most natural framework to account for the smallness of neutrino masses.

We point out, in this paper, that the seesaw-induced mass terms for neutrinos generate instability of the vacuum in the supersymmetric(SUSY) standard model, if the anomaly mediation [3,4] of SUSY breaking provides the dominant contribution to all the SUSY-standard-model fields. The transition of our false vacuum to the true vacuum is strongly suppressed at zero temperature, since the true minimum is separated far away from our false vacuum. However, the thermal transition seems to be effective at high temperatures because of the thermal excitations, which may lead to horrible universes. We find, contrary to the above thought, that the thermal transition is also strongly suppressed and hence the anomaly mediation models are perfectly consistent with the present observation. Therefore, the reheating temperature after inflation would not be constrained from above in anomaly mediation models with the seesaw-induced mass terms for neutrinos. This our observation makes the anomaly mediation models very attractive, since most of baryo & lepto-genesis models need sufficiently high temperature  $T_R \gtrsim 10^{10} \text{ GeV}$  [5]. Notice that the anomaly mediation models predict the large gravitino mass  $m_{3/2} \simeq 100 \text{ TeV}$  avoiding the “gravitino problem”, which is a serious problem in gravity-mediation models [6].

## II. SEESAW MECHANISM IN ANOMALY MEDIATION MODEL

In the anomaly mediation models the SUSY breaking in the hidden sector is transmitted to the observed sector by super-Weyl anomaly effects [3,4]. In particular, the anomaly effects provide the dominant contribution to gaugino masses as

$$m_{G_a} = \frac{b_a g_a^2}{16\pi^2} \langle \Phi \rangle |_{\theta^2} \quad (a = 1, 2, 3), \quad (1)$$

where  $g_a$  ( $a = 1, 2, 3$ ) are the gauge coupling constants for  $U(1)_Y, SU(2)_L$  and  $SU(3)_C$  gauge groups, and  $b_a$  ( $a = 1, 2, 3$ ) are  $\beta$ -function coefficients of the corresponding gauge coupling constants.  $\Phi$  is the supergravity auxiliary supermultiplet whose  $\theta^2$  component's vacuum-expectation value (vev) is of order the gravitino mass  $m_{3/2}$ . It is very crucial in the present analysis that the gaugino masses are one-loop suppressed relative to the gravitino mass and hence the gravitino is extremely heavy. For the gluino mass  $m_{G_3} \simeq 1 \text{ TeV}$  we see the gravitino mass  $m_{3/2} \simeq 100 \text{ TeV}$ . The SUSY-breaking (mass)<sup>2</sup> for scalar bosons are induced at the two-loop level. However, pure anomaly mediation predicts slepton (mass)<sup>2</sup> to be negative, requiring additional contributions to the slepton masses. In the present analysis we employ the simple phenomenological solution [7] to this problem that merely adds a universal mass term  $m_0^2$  of order the electroweak scale  $(100 \text{ GeV} - 1 \text{ TeV})^2$  to all of the scalar masses, leaving the gaugino and gravitino masses unchanged. We assume, in this paper, the gravitino mass  $m_{3/2} \simeq 100 \text{ TeV}$ .

Let us now discuss the seesaw-induced mass terms for neutrinos. The integration of heavy right-handed neutrinos generates the following non-renormalizable operator in the low-energy superpotential:

$$W = \frac{1}{M_{Ri}} (L_i H_u)^2, \quad (2)$$

where  $L_i$  ( $i = 1, 2, 3$ ) and  $H_u$  are chiral supermultiplets for lepton and Higgs doublets and  $i$  denotes family index.  $M_{Ri}$  represent the effective Majorana masses for right-handed neutrinos. We take, in the present analysis,  $M_{R3} \simeq (0.3 - 1) \times 10^{15}$  GeV, reproducing  $m_{\nu_3} \simeq (0.3 - 1) \times 10^{-1}$  eV suggested from the Superkamiokande experiments on the atmospheric neutrino oscillation [1]. We suppress the family indices in the following discussion.

We consider the following  $D$ -term flat direction:

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad (3)$$

and others = 0. Then, the superpotential is written as

$$W = \frac{1}{4M_R} \phi^4. \quad (4)$$

The scalar potential is given by

$$V = m_\phi^2 |\phi|^2 - \frac{m_{3/2}}{4M_R} (\phi^4 + h.c.) + \left| \frac{1}{M_R} \phi^3 \right|^2, \quad (5)$$

where  $m_\phi^2$  is the soft SUSY breaking mass for  $\phi$ . Note that we have assumed our desired vacuum  $\langle \phi \rangle = \phi_F \equiv 0$  to have an (almost) vanishing cosmological constant. We easily see that the desired minimum  $\langle \phi \rangle = 0$  is no longer the absolute minimum in the theory of anomaly mediation with  $m_{3/2} \simeq 100m_\phi$  much larger than  $m_\phi$ . The true vacuum appears at<sup>1</sup>

$$\langle \phi \rangle \simeq \phi_T \equiv \sqrt{\frac{m_{3/2} M_R}{3}} \sim 10^{10} \text{ GeV}, \quad (6)$$

and it has a negative cosmological constant given by

$$V_{\text{true}} = -\frac{m_{3/2}^3 M_R}{54}. \quad (7)$$

Since the potential energy  $V(\phi = 0) \simeq 0$  of the present universe is much higher than  $V_{\text{true}}$ , we live in the false vacuum now<sup>2</sup>.

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<sup>1</sup>This (true) vacuum can not be identified with the vacuum we live in, since the electroweak gauge bosons are too heavy as  $m_W \simeq m_Z \sim 10^{10}$  GeV there.

<sup>2</sup> It may be possible that the  $H_u = L$  flat direction is lifted up in some extended models by introducing new particles at intermediate energy scales. In those cases, the true vacuum with a large negative cosmological constant may not appear.

This false vacuum is in principle unstable against tunneling into the true vacuum. If the anomaly mediated SUSY breaking models really describe our world, the tunneling rate should be very small in order that the present false vacuum survives at least longer than the age of the present universe. Furthermore, we must have had some certain cosmological history that has safely led us to this false vacuum.

### III. VACUUM TRANSITION RATE

Let us estimate the tunneling rate of our false vacuum. At zero temperature we must estimate four dimensional Euclidean action  $S_4$  evaluated with the bounce solution for the potential (5) [8]. Since the potential at the true minimum is deep in comparison with the height of the potential barrier [*i.e.*  $V_{\text{barrier}} \sim (m_\phi/m_{3/2})^4 |V_{\text{true}}| \ll |V_{\text{true}}|$ ], we can neglect the  $\phi^6$ -term in the potential for  $|\phi| \lesssim \phi_{\text{barrier}} \sim \sqrt{(m_\phi/m_{3/2})} \phi_T$  (*i.e.* the thick wall approximation) [see Fig. 1]. Then  $S_4$  is given by

$$S_4 = \int dx^4 \left( \frac{1}{2} \left( \frac{d\varphi}{dt} \right)^2 + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi) \right), \quad (8)$$

$$V(\varphi) \simeq \frac{1}{2} m_\phi^2 \varphi^2 - \frac{1}{8} \frac{m_{3/2}}{M_R} \varphi^4, \quad (9)$$

where  $\varphi = \text{Re}\phi/\sqrt{2}$  and  $\varphi$  satisfies

$$d^2\varphi/dt^2 + \nabla^2\varphi = dV/d\varphi. \quad (10)$$

By redefining  $\varphi = m_\phi(4M_R/m_{3/2})^{1/2}\psi$ ,  $x_\mu = \xi_\mu/m_\phi$  with  $\psi$  and  $\xi$  being dimensionless, we can easily see that  $S_4$  is rewritten as

$$S_4 = \frac{4M_R}{m_{3/2}} S_4(\psi), \quad (11)$$

$$S_4(\psi) \simeq \int d^4\xi \left( \frac{1}{2} (d\psi/d\xi^0)^2 + \frac{1}{2} (\nabla_\xi \psi)^2 + \frac{1}{2} \psi^2 - \frac{1}{2} \psi^4 \right), \quad (12)$$

where  $S_4(\psi) \sim 10$  [9]. Then we obtain the tunneling rate  $\Gamma_4$  as

$$\Gamma_4 \sim m_\phi^4 S_4^2 \exp(-S_4) \sim m_\phi^4 \frac{1600 M_R^2}{m_{3/2}^2} \exp\left(-\frac{40 M_R}{m_{3/2}}\right). \quad (13)$$

Since  $M_R/m_{3/2} \sim 10^{10}$ , the tunneling rate  $\Gamma_4$  is negligibly small and hence the vacuum transition into the true vacuum is sufficiently suppressed at zero temperature.

We can see that this stability against the tunneling comes from the smallness of the coefficient  $\lambda$  of the quartic term ( $\lambda \equiv 1/8(m_{3/2}/M_R) \simeq 10^{-11} \ll 1$ ) in the potential (9). Indeed the exponent is roughly estimated as

$$S_4 \sim (4\text{D bubble volume}) \times (\text{Lagrangian}) \sim \frac{1}{m_\phi^4} V_{\text{barrier}} \quad (14)$$

where

$$V_{\text{barrier}} \sim \frac{1}{\lambda} m_\phi^4 \gg m_\phi^4, \quad (15)$$

and hence

$$S_4 \sim \frac{1}{\lambda} \left( = \frac{M_R}{m_{3/2}} \right) \gg 1. \quad (16)$$

The small coefficient has led to the large expectation value  $\phi_T \simeq (1/\sqrt{\lambda})m_\phi$  and the large potential barrier compared with the mass  $m_\phi$ .

We have found that once the  $\langle \phi \rangle = \phi_F \equiv 0$  vacuum (which we call  $\phi_F$ -vacuum hereafter) was chosen and the temperature has dropped down almost to zero, the quantum tunneling to the  $\langle \phi \rangle = \phi_T$  vacuum (which we call  $\phi_T$ -vacuum) is sufficiently suppressed. Now let us examine whether or not this  $\phi_F$ -vacuum is chosen naturally in the cosmological history.

What can be plausibly taken to be the initial condition of the cosmological history? We have many evidences which suggest that there exists an inflationary epoch before big bang nucleosynthesis (BBN). This inflationary epoch is followed by reheating process. The energy density of the universe which was initially carried by the inflaton potential energy is gradually but completely converted into thermal plasma energy through inflaton decay.

During this reheating process, the temperature of the thermal plasma changes as a function of time as [10]

$$T^4 = 1.2 H \Gamma_{\text{inf}} M_G^2, \quad H = \frac{2}{3t}, \quad (17)$$

where  $H$  is Hubble parameter during the reheating process,  $\Gamma_{\text{inf}}$  decay rate of the inflaton and  $M_G \simeq 2.4 \times 10^{18}$  GeV the reduced Planck mass. The universe takes the maximum temperature  $T_m$  soon after inflation and this temperature is much higher than that at the end of the reheating process (the reheating temperature  $T_R \sim (\Gamma_{\text{inf}}^2 M_G^2)^{1/4}$ ). In this thermal plasma with high temperature, the  $\phi$  field feels finite temperature effective potential which is drastically different from the zero temperature potential (5).

Now let us see this thermal effective potential in detail. Particles which have interactions with the  $\phi$  field give contributions to the thermal potential of the  $\phi$  field. These contributions exist as long as those particles are in thermal equilibrium and therefore as long as masses of those particles are less than the temperature. The  $\phi$  field expectation value  $\langle \phi \rangle$  gives masses to those particles through the superpotential,

$$W = Y_i Q_i \bar{U} \frac{\phi}{\sqrt{2}} + Y_{\text{eff}} \bar{E}_{\text{eff}} H_d \frac{\phi}{\sqrt{2}}, \quad (18)$$

$$Y_{\text{eff}} = \sqrt{\sum_f |Y_f U_{f3}|^2}, \quad (19)$$

$$\bar{E}_{\text{eff}} = \frac{1}{\sqrt{\sum_f |Y_f U_{f3}|^2}} \sum_f Y_f U_{f3} \bar{E}_f, \quad (20)$$

where  $Y_i$  denote up-type Yukawa couplings,  $Y_f$  denote charged-lepton Yukawa couplings,  $U_{f3}$  denote lepton-flavor mixing matrix elements, and  $H_d$  is Higgs supermultiplet couple to down-type quarks.  $Y_i$  and  $Y_f$  are given by

$$Y_i = m_i/(174\text{GeV} \sin \beta) \quad (i = \text{top, charm, up}), \quad (21)$$

$$Y_f = m_f/(174\text{GeV} \cos \beta) \quad (f = \tau, \mu, e), \quad (22)$$

where  $\beta = \arctan(\langle H_u \rangle / \langle H_d \rangle)$ . Masses of  $Q_i$  and  $\bar{U}_i$  supermultiplet particles are  $Y_i \phi / \sqrt{2}$  and those of  $\bar{E}_{\text{eff}}$  and  $H_d$  particles are  $Y_{\text{eff}} \phi / \sqrt{2}$ . Hence the range of the field value in which the thermal potentials arise from those particle loops are roughly limited to  $Y|\varphi| \lesssim T$ . The contribution to the thermal potential (= free energy) from each chiral multiplet is given by  $-T^4 \pi^2 / 24 + Y^2 \varphi^2 T^2 / 8$ .

We now assume, for example,  $T_m > 10^{10}\text{GeV}$ . In this case, while the temperature is higher than  $10^{10}\text{GeV}$ , the top quark multiplet does not decouple from the thermal equilibrium even for  $\varphi \sim \phi_T \simeq \sqrt{M_R m_{3/2}} \sim 10^{10}\text{GeV}$  (since  $m_{\text{top}} \simeq Y_t \phi_T < T$ ).<sup>3</sup> Then for all  $\varphi \lesssim \phi_T$ , the thermal mass term  $(1/2)m_{\text{eff}}^2 \varphi^2$  arising from top (s)quark multiplet, where  $m_{\text{eff}} \sim Y_t T \gtrsim 10^{10}\text{GeV} \gg m_{3/2}$ , is dominant over the negative quartic term which originally exists. There is no local minimum anywhere except for  $\varphi = \phi_F = 0$  and the universe sits around the origin  $\varphi = 0$ .

When the temperature drops below  $10^{10}\text{GeV}$ , the top (s)quark multiplet decouples from the equilibrium for  $\varphi \sim \mathcal{O}(\phi_T)$  and this multiplet contributes to the thermal potential only for  $\varphi \ll \phi_T$ . While the temperature is larger than  $10^8\text{GeV}$  (and less than  $10^{10}\text{GeV}$ ), the effective mass term for  $\varphi \sim \mathcal{O}(\phi_T)$  comes from the thermal contribution of lepton, Higgs and charm (s)quark multiplets. This thermal mass term is also large enough to dominate the negative quartic term (since  $m_{\text{eff}} \sim Y_{c,\tau} T \gtrsim 10^6\text{GeV} \gg m_{3/2}$ ) and hence the  $\varphi = 0$  minimum is still the only and absolute minimum.<sup>4</sup>

After the temperature gets down below  $10^8\text{GeV}$ , lepton, Higgs and charm (s)quark supermultiplets no longer give thermal potential for  $\varphi \sim \mathcal{O}(\phi_T) \sim \mathcal{O}(10^{10}\text{GeV})$ , since the temperature is not enough to thermalize those particles with masses  $m_{c,H_d,\bar{E}_{\text{eff}}} \sim Y_{c,\tau} \phi_T \sim 10^8\text{GeV}$ . The thermal potential by now comes only from up (s)quark supermultiplet,<sup>5</sup> which is so tiny (since  $m_{\text{eff}} \simeq \sqrt{(Y_u T)^2 + m_\phi^2} \simeq m_\phi \ll m_{3/2}$ ) that the effective potential of the field  $\phi$  begins to show a dip around the would-be true minimum  $\varphi \simeq \phi_T \simeq 10^{10}\text{GeV}$  (Fig.2). As the temperature falls further, this dip gets larger. The local minimum newly appeared there will become the true minimum before the temperature drops down to  $10^7\text{GeV}$  (Fig.2).

This change in the shape of effective thermal potential is exactly the same as that in the standard first order phase transition. Naive guess will tell us that the phase transition to the  $\phi_T$ -vacuum occurred in the history of the universe as usual. We study in the following whether the phase transition really occurs or not.

First order phase transitions are known to take place through following two mechanisms. One mechanism is through equilibrium between  $\phi_F$ - and  $\phi_T$ -vacua [11], and the other is the conventional bubble nucleation process [9].

<sup>3</sup>Here and from now on, we assume that  $Y_t \sim 1$ ,  $Y_{c,\tau} \sim 10^{-2}$  and  $Y_u \sim 10^{-4}$ .

<sup>4</sup>We notice here that we do not need to assume  $T_m > 10^{10}\text{GeV}$ . Assumption of  $T_m > 10^8\text{GeV}$  is sufficient.

<sup>5</sup>Electroweak gauge bosons have already decoupled from the thermal equilibrium at this stage.

At first we study the phase transition through the thermal equilibrium between  $\phi_F$ -vacuum ( $\langle\phi\rangle = 0$ ) and  $\phi_T$ -vacuum ( $\langle\phi\rangle \simeq \phi_T \sim 10^{10}\text{GeV}$ ). Key idea of this mechanism is as follows. Once the dip in the potential is formed then domains of  $\phi_F$ -vacua and  $\phi_T$ -vacua appear. The size of each domain is roughly the correlation length ( $\sim$  curvature inverse) of the scalar field. These two types of domains are in the thermal equilibrium via thermal transition of vacua beyond the potential barrier. The transition rate from  $\phi_F$ -vacuum to  $\phi_T$ -vacuum and vice versa are given by [11]

$$\Gamma_{F\rightarrow T} \sim m_F^4 e^{-\frac{F_{F\rightarrow T}}{T}} \quad (23)$$

$$\Gamma_{F\leftarrow T} \sim m_T^4 e^{-\frac{F_{F\leftarrow T}}{T}} \quad (24)$$

where  $m_{F,T}$  are curvature of the effective potential at each vacua,  $F_{F\rightarrow T, F\leftarrow T}$  are free energy barrier of transition in each direction. The ratio of number of  $\phi_F$ - and  $\phi_T$ - vacua is determined by detailed balance condition if the system is in the equilibrium:

$$N_F \Gamma_{F\rightarrow T} = \Gamma_{F\leftarrow T} N_T \quad (25)$$

$$\frac{N_T}{N_F} = \left(\frac{m_F}{m_T}\right)^4 \exp\left(-\frac{F_{F\rightarrow T} - F_{F\leftarrow T}}{T}\right). \quad (26)$$

If the system keeps equilibrium until the  $\phi_T$ -vacuum really becomes true vacuum and until the domain size becomes larger than the critical radius,<sup>6</sup> then the true ( $\phi_T$ -)vacuum domains begin to expand and percolate, and the phase transition is completed.

Now let us examine if the equilibrium is maintained for the case of our interest. We are going to make an estimate of the transition rate from the  $\phi_F$ - to the  $\phi_T$ -vacuum. When the dip in the potential was formed ( $T \sim 10^8\text{GeV}$ ), the barrier height of the free energy density in this transition is roughly given by

$$f_{F\rightarrow T} \sim Y_{c,\tau}^2 T^2 \phi_T^2. \quad (27)$$

Since the typical volume of each domain is roughly  $m_{\text{eff}}^{-3} \simeq (1/(Y_{c,\tau}T)^3)$ , the barrier of the free energy is written as

$$F_{F\rightarrow T} = \frac{Y_{c,\tau}^2 T^2 \phi_T^2}{(Y_{c,\tau}T)^3}. \quad (28)$$

Then, we obtain the transition rate as

$$\Gamma_{F\rightarrow T} \sim (Y_{c,\tau}T)^4 \exp\left(-\frac{\phi_T^2}{Y_{c,\tau}T^2}\right) \sim 10^{-8}T^4 \exp(-\text{factor} \times 10^5). \quad (29)$$

This transition rate is extremely small compared with the expansion rate of the universe  $H^4 \gtrsim T^8/M_G^4 \sim T^4 10^{-40}$ . Therefore, it is concluded that the transition between the  $\phi_F$ -vacuum and the  $\phi_T$ -vacuum is already frozen out, and hence no  $\phi_T$ -vacuum domain is

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<sup>6</sup>Characterization of the “critical radius” is as follows: Bubbles with radius larger than the “critical radius” expand and others not.

created. The phase transition through the equilibrium between  $\phi_F$ - and  $\phi_T$ -vacuum does not take place in the case of our interest.

Even if the thermal equilibrium of two vacua are not formed, the false( $\phi_F$ -)vacuum can decay into true ( $\phi_T$ -)one, after the potential energy of the  $\phi_T$ -vacuum becomes lower than that of the  $\phi_F$ -one, through the thermal bubble nucleation. We estimate the thermal transition rate below. In the case of finite temperature, the transition rate is determined by three dimensional Euclidean action  $S_3$  [9] which is given by

$$S_3 = \int d^3x \left( \frac{1}{2} (\nabla \varphi)^2 + V(\varphi)_{\text{thermal}} \right), \quad (30)$$

where  $V(\varphi)_{\text{thermal}}$  is the finite temperature potential and  $\varphi$  satisfies

$$\nabla^2 \varphi = dV_{\text{thermal}}/d\varphi. \quad (31)$$

In the same way as  $S_4$ ,  $S_3$  is calculated with use of the dimensionless quantities [ $\varphi = m_{\text{eff}}(4M_R/m_{3/2})^{1/2}\psi$ ,  $x_\mu = \xi_\mu/m_{\text{eff}}$ ] as

$$S_3 = \frac{4m_{\text{eff}}M_R}{m_{3/2}} S_3(\psi), \quad (32)$$

$$S_3(\psi) \simeq \int d^3\xi \left( \frac{1}{2} (\nabla_\xi \psi)^2 + \frac{1}{2} \psi^2 - \frac{1}{2} \psi^4 \right), \quad (33)$$

where  $S_3(\psi) \simeq 9.5$  [9]. Then the transition rate with temperature  $T$  is estimated as

$$\begin{aligned} \Gamma_3 &\simeq T^4 \left( \frac{S_3}{2\pi T} \right)^{3/2} \exp(-S_3/T) \\ &\simeq T^4 \left( \frac{19m_{\text{eff}}M_R}{2\pi T m_{3/2}} \right)^{3/2} \exp\left( -\frac{19m_{\text{eff}}M_R}{T m_{3/2}} \right). \end{aligned} \quad (34)$$

As mentioned before, the  $\phi_T$ -vacuum becomes really true vacuum after the temperature cools down to  $\sim 10^7$  GeV. At that time,  $m_{\text{eff}}$  is already  $\simeq m_\phi$ . Therefore, the exponent is bounded from below as  $10^7$  for  $T \lesssim 10^7$  GeV.

Now let us calculate the fraction  $P(t_0)$  in the present universe which remains in the  $\phi_F$ -vacuum state. Assuming that the  $\phi_T$ -vacuum bubble produced in the transition expands at the light velocity, we obtain  $P(t_0)$  [12,9],

$$P(t_0) = \exp \left[ - \int_{t_i}^{t_0} dt_1 \Gamma_3(t_1) a(t_1)^3 \left\{ \frac{4\pi}{3} \left( \int_{t_1}^{t_0} \frac{dt_2}{a(t_2)} \right)^3 \right\} \right], \quad (35)$$

where  $t_i$  is the initial time. We assume, for simplicity, that the critical temperature below which the  $\phi_T$ -bubbles are formed is lower than the reheating temperature, or in other words, the thermal bubble nucleation occurs in the radiation dominated universe<sup>7</sup>. Then,

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<sup>7</sup>This is mere a technical assumption and is not important.



$$a(t_1)^3 \left\{ \frac{4\pi}{3} \left( \int_{t_1}^{t_0} \frac{dt_2}{a(t_2)} \right)^3 \right\} \sim \left( \frac{T_0}{T_1} \right)^3 \frac{1}{H_0^3}, \quad (36)$$

where  $T_0$  and  $H_0$  are the temperature and Hubble parameter in the present universe, respectively. Since the transition rate  $\Gamma_3$  decreases rapidly as the temperature falls, the time integration in the eq.(35) can be replaced by

$$dt_1 \sim d\left(\frac{M_G}{T^2}\right) \sim d\left(\frac{1}{T}\right) \frac{M_G}{T} \sim \frac{1}{S_3} \frac{M_G}{T_i}, \quad (37)$$

where the  $T_i$  is the initial temperature  $10^7 \text{ GeV}$ . The total expression of the eq.(35) is

$$\begin{aligned} P(t_0) &\sim \exp \left[ - \left( \frac{T_0}{H_0} \right)^3 \frac{M_G}{S_3} \left( \frac{S_3}{2\pi T_i} \right)^{\frac{3}{2}} \exp \left( -\frac{S_3}{T_i} \right) \right] \\ &\sim \exp \left[ -10^{87} \frac{M_G}{S_3} \left( \frac{S_3}{2\pi T_i} \right)^{\frac{3}{2}} \exp \left( -\frac{S_3}{T_i} \right) \right]. \end{aligned} \quad (38)$$

$P(t_0)$  is almost 1 if the exponent  $S_3/T_i$  is larger than 200. Therefore the condition that the most of our universe is in the  $\phi_F$ -vacuum today (*i.e.*  $1 - P(t_0) \ll 1$ ) requires  $S_3/T_i \gtrsim 200$ , which is satisfied in the present case ( $S_3/T_i \sim 10^7$ ).

#### IV. CONCLUSION

We have studied the vacuum stability in the anomaly mediated SUSY breaking models with massive neutrinos. If the small masses of neutrinos are generated by the seesaw mechanism, the seesaw-induced mass terms make the present vacuum ( $\phi_F$ -vacuum) with an (almost) vanishing cosmological constant unstable and the true vacuum ( $\phi_T$ -vacuum) has a disastrously large negative cosmological constant.

Although our false vacuum has quite high energy density compared with the true vacuum, the quantum transition into the true vacuum is highly suppressed. High temperature (more than  $10^8 \text{ GeV}$ ) thermal plasma that is created after inflation gives effective potential of the field  $\phi$  with a unique vacuum at the origin  $\langle \phi \rangle = 0$  ( $\phi_F$ -vacuum). As the temperature decreases the effective potential shows a new local minimum ( $\phi_T$ -vacuum) which turns into the true minimum at  $T \lesssim 10^7 \text{ GeV}$ . The phase transition to the  $\phi_T$ -vacuum is also highly suppressed both through the thermal equilibrium domain formation of both vacua and through thermal tunneling decay process. Thus, we can live on a supercooled false vacuum state.

Since the thermal transition from the  $\phi_F$ -vacuum to the  $\phi_T$ -vacuum is negligible, it does not give any constraint on the reheating temperature after inflation. Furthermore, the gravitino mass is so heavy ( $\simeq 100 \text{ TeV}$ ) in anomaly mediation models that gravitinos decay much earlier than the BBN and hence the model avoids the “gravitino problem”, which is a serious problem in gravity-mediation models. Thus, the reheating temperature is not constrained from above, which is very favored by many baryogenesis scenarios.

We have discussed only on the heaviest neutrino direction. For lighter neutrino directions the analyses are the same. We easily see that the potential barrier between the true minimum and our false vacuum is much higher and the tunneling into the true vacuum is much more suppressed.

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FIGURES

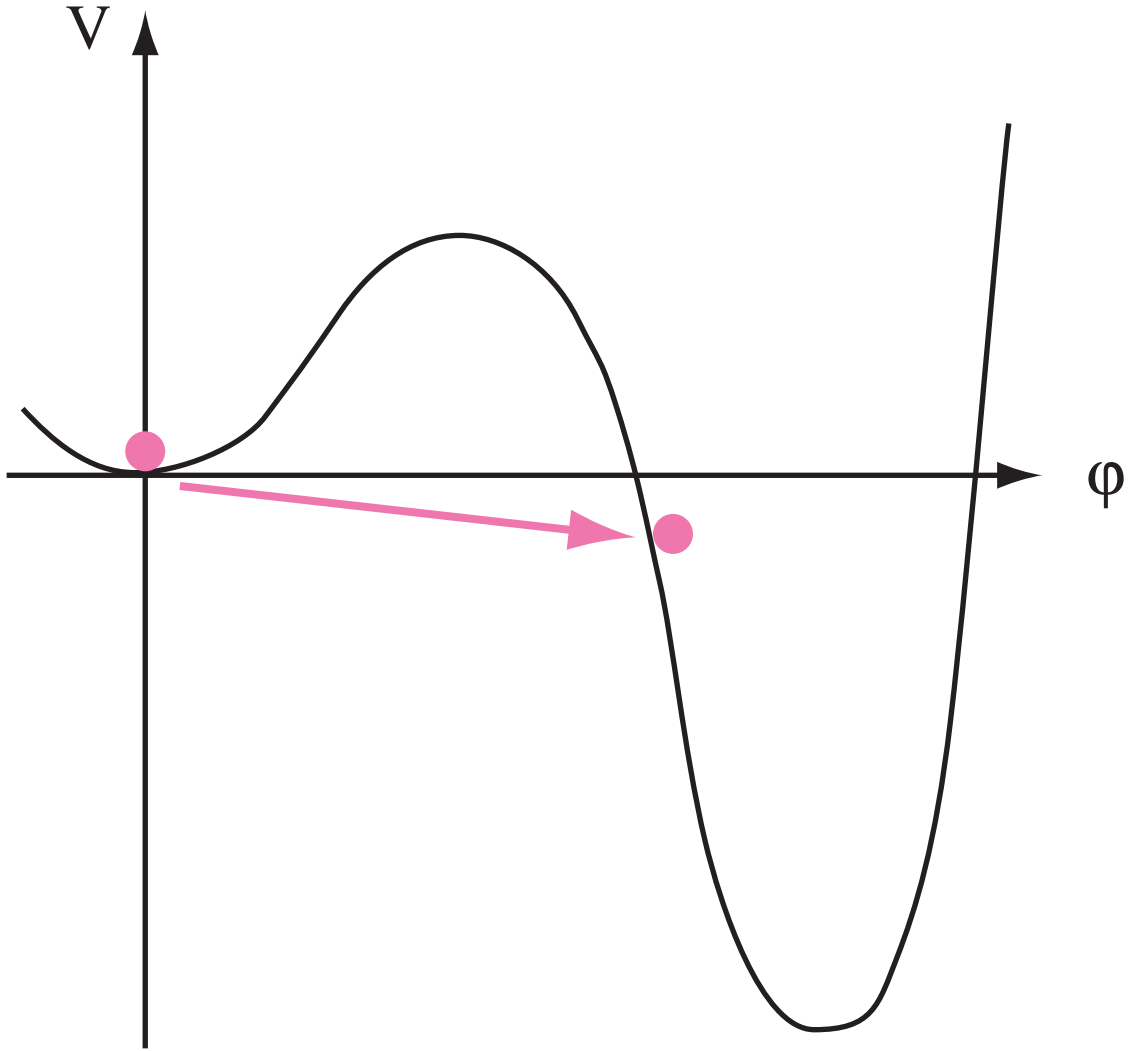


FIG. 1. Potential for  $\varphi$ .

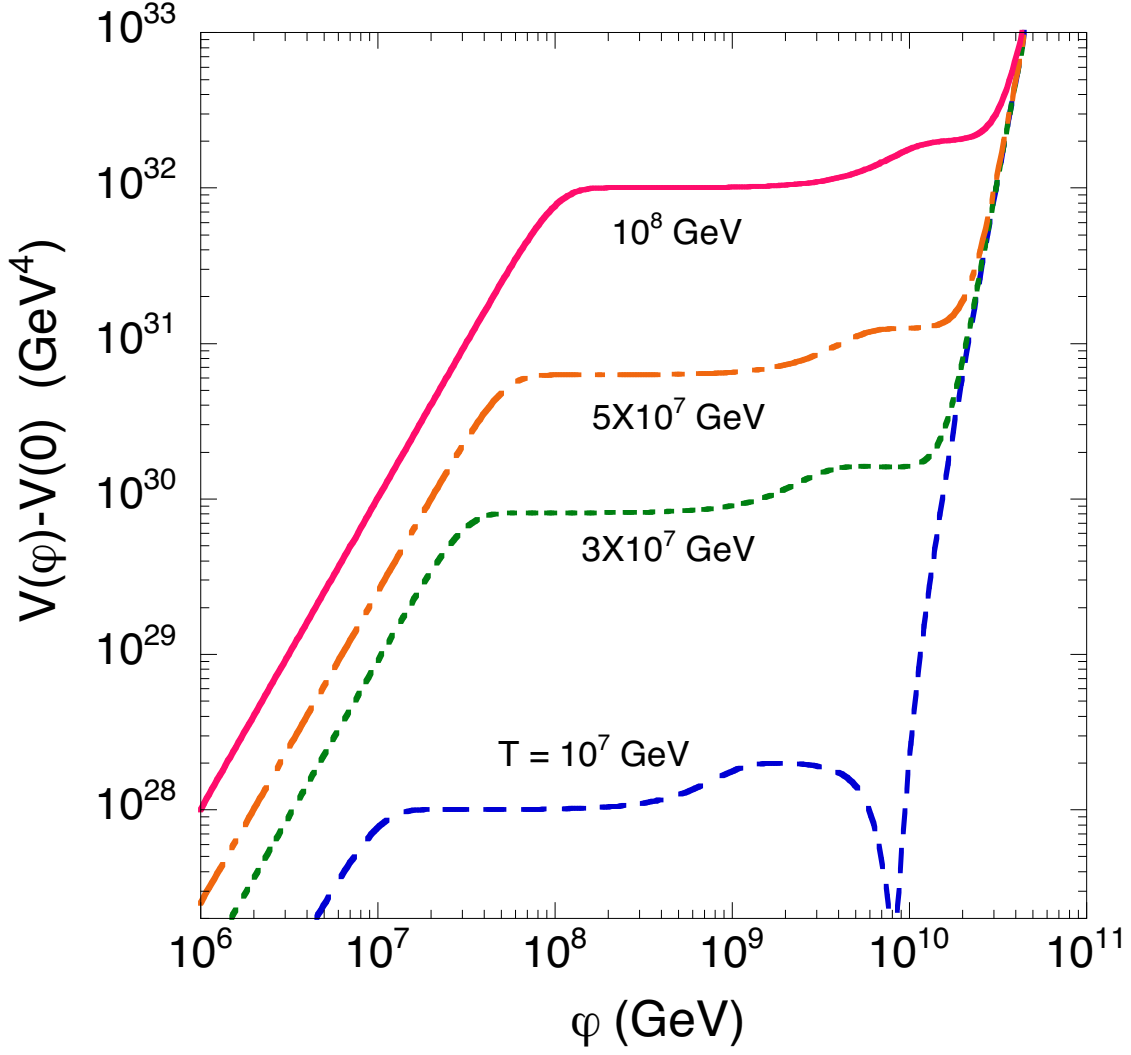


FIG. 2. Finite temperature potential for  $\varphi$ . The solid, dash-dotted, short dashed and long dashed lines represent the potentials for temperature  $10^8, 5 \times 10^7, 3 \times 10^7$  and  $10^7$  GeV, respectively.